Homework 4 Solution

EE 451/551 Wind Energy

Due 11:00 pm, Feb. 7, 2019. Submit online at
`https://www.dropbox.com/request/IJJdakpNj6RqCaIGcGEr`

Problem 1. A 6-pole, 60 Hz, induction generator is receiving 500 hp from the blades and is rotating at 1250 r/min. The rotational loss of the generator is 1 kW. Find the slip of the generator and its developed power.

Solution. Recall that the synchronous speed of an induction generator is related to the number of poles by

$$n_s = 120f/p = 120(60/6) = 1200\,\text{r/min},$$

where f denotes the nominal grid frequency. Hence, the slip is

$$s = \frac{n_s - n}{n_s} = \frac{1200 - 1250}{1200} = -0.042.$$

The problem states that the input power from the prime mover is 500 hp, or 372.849 kW. Recall that the relationship between the input power, the developed power, and the rotational losses is given by

$$P_{\text{in}} = P_{\text{rot}} + P_d.$$

Hence, the developed power is

$$P_d = P_{\text{in}} - P_{\text{rot}} = 371.849\,\text{kW},$$

where $P_{\text{rot}} = 1\,\text{kW}$.

Problem 2. A 690 V induction generator has the following parameters: $x_1 = 2\,\Omega$; $x_2' = 1.5\,\Omega$; $x_m = 100\,\Omega$; $r_1 = 1\,\Omega$; $r_2' = 1\,\Omega$. The slip is $s = -0.1$. Using the Thévenin equivalent circuit for the grid voltage, find:

a) The developed power.

Solution. Applying the voltage divider rule to solve for V_{th} yields

$$V_{\text{th}} = \left(\frac{jX_m}{Z_1 + jX_m}\right) V_1 = 390.541\angle 0.562^\circ\,\text{V}.$$

The corresponding Thévenin impedance is given by

$$Z_{\text{th}} = \left(\frac{1}{Z_1} + \frac{1}{jX_m}\right)^{-1} = \frac{Z_1 jX_m}{Z_1 + jX_m} = 0.961 + j1.970\,\Omega.$$

The developed power is then

$$P_d = -3|I_2'|^2 R_2' \left(\frac{1-s}{s}\right) = \frac{-3|V_{\text{th}}|^2 R_2'(1-s)}{s\left[(R_{\text{th}} + R_2'/s)^2 + X_{\text{eq}}^2\right]} = 53.691\,\text{kW},$$

where $X_{\text{eq}} = X_{\text{th}} + X_2'$.

b) The active and reactive power delivered to the grid.

Solution. The 3-phase complex power delivered to the grid can be expressed as

$$S_{\text{out}} = 3V_1 I_1{}^*,$$

where I_1 is the stator current. Recall that

$$I_1 = \frac{V_m - V_1}{R_1 + jX_1} = 41.139\angle 27.191^\circ\,\text{A},$$

where

$$V_m = V_D - I_2'\,(R_2' + X_2') = -I_2'\left(\frac{R_2'}{s} + X_2'\right) = 407.874\angle 13.034^\circ\,\text{V}.$$

Solving for the active and reactive power delivered to the grid, we have

$$\begin{aligned} P_{\text{out}} &= \Re\left\{3V_1 I_1{}^*\right\} = 43.733\,\text{kW} \\ Q_{\text{out}} &= \Im\left\{3V_1 I_1{}^*\right\} = -22.467\,\text{kVAr}, \end{aligned}$$

where the negative sign indicates that the machine is consuming reactive power. The difference between P_{out} and P_d stems from the copper losses.

Problem 3. A wind turbine has a 6-pole, 690 V induction generator with the following parameters: $r_1 = r_2' = 0.05\,\Omega$ and $x_1 = x_2' = 0.5\,\Omega$. Ignore the shunt reactance. The generator is rotating at 1230 r/min. Compute:

a) The developed power.

Solution. Since we can ignore the magnetizing reactance, the Thévenin equivalent is straightforward

$$\begin{aligned} V_{\text{th}} &= V_1 = \left(690/\sqrt{3}\right)\angle 0^\circ \\ Z_{\text{th}} &= Z_1 = 0.05 + j0.5\,\Omega. \end{aligned}$$

Solving for the developed power, we have

$$P_d = \frac{-3|V_{\text{th}}|^2 R_2'(1-s)}{s\left[(R_{\text{th}} + R_2'/s)^2 + X_{\text{eq}}^2\right]} = 203.229\,\text{kW}.$$

b) The developed torque.

Solution. Recall that the developed torque and power are related by $T_d = P_d/\omega$. Hence,

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(1230)}{60} = 128.805\,\text{rad/s}$$
$$T_d = \frac{P_d}{\omega} = 1577.796\,\text{N m}.$$

c) The copper losses.

Solution. Recall that the copper losses are related to the developed power and the output power so that

$$P_{\text{out}} = P_d - P_{\text{cu1}} - P_{\text{cu2}}.$$

Thus, the sum of the copper losses in the rotor and stator is given by

$$P_{\text{cu1}} + P_{\text{cu2}} = P_d - P_{\text{out}}$$
$$P_{\text{cu1}} + P_{\text{cu2}} = P_d - \Re\left\{3V_1 {I_1}^*\right\} = 9.914\,\text{kW}.$$

Alternatively, we could have also found the copper losses directly as

$$P_{\text{cu1}} + P_{\text{cu2}} = 3|I_1|^2 R_1 + 3|I_2'|^2 R_2' = 9.914\,\text{kW}.$$

d) The output power.

Solution. As in Problem 2, we can find the output power using the relation

$$P_{\text{out}} = \Re\left\{3V_1 {I_1}^*\right\} = 193.314\,\text{kW}.$$

Problem 4. A type 1 wind turbine has a 6-pole, 60 Hz induction generator connected to a grid voltage of 690 V. The speed of the generator is 1250 r/min. The parameters are $r_1 = r_2' = 10\,\text{m}\Omega$ and $x_1 = x_2' = 100\,\text{m}\Omega$. Ignoring the shunt reactance, find:

a) The developed power and torque.

Solution. Since we are neglecting the magnetizing reactance, the Thévenin equivalent is simply

$$V_{\text{th}} = V_1 = \left(690/\sqrt{3}\right)\angle 0°$$
$$Z_{\text{th}} = Z_1 = 0.01 + j0.1\,\Omega.$$

Solving for the developed power, we have

$$P_d = \frac{-3|V_{\text{th}}|^2 R_2'(1-s)}{s\left[(R_{\text{th}} + R_2'/s)^2 + X_{\text{eq}}^2\right]} = 1.281\,\text{MW},$$

where $X_{\text{eq}} = X_{\text{th}} + X_2'$. The corresponding developed torque is

$$T_d = \frac{P_d}{\omega} = 9787.772\,\text{N m},$$

where $\omega = 2\pi n/60$.

b) The maximum possible power and torque.

Solution. The slip at maximum power follows the expression

$$m = \frac{R_2'}{\sqrt{R_{\text{th}}^2 + 2R_{\text{th}}R_2' + X_{\text{eq}}^2}}$$
$$\hat{s} = -m\left(m + \sqrt{m^2+1}\right) = -0.0524.$$

Inserting this slip to solve for the maximum developed power, we have

$$P_d = \frac{-3|V_{\text{th}}|^2 R_2'(1-\hat{s})}{\hat{s}\left[(R_{\text{th}} + R_2'/\hat{s})^2 + X_{\text{eq}}^2\right]} = 1.315\,\text{MW}.$$

The slip at maximum torque follows the expression

$$s_\star = \frac{-R_2'}{\sqrt{R_{\text{th}}^2 + X_{\text{eq}}^2}} = -0.0499.$$

The corresponding maximum torque is then

$$T_d = \frac{-3|V_{\text{th}}|^2 R_2'}{s_\star \omega_s \left[\left(R_{\text{th}} + R_2'/s_\star\right)^2 + X_{\text{eq}}^2\right]} = 9957.126\,\text{N m},$$

where we have used the definition of the slip to make the substitution

$$s_\star = \frac{\omega_s - \omega_\star}{\omega_s} \implies \frac{1}{\omega_s} = \frac{1-s_\star}{\omega_\star}.$$

Problem 5. A type 1 wind turbine has a 6-pole, 60 Hz induction generator connected to a grid voltage of 415 V. The speed of the generator is 1260 r/min. The parameters are $r_1 = r_2' = 4\,\text{m}\Omega$, $x_1 = x_2' = 40\,\text{m}\Omega$ and $x_m = 5\,\Omega$. Using Thévenin equivalent circuit for the grid, find the reactive power delivered by the grid to the generator.

Solution. Applying the voltage divider rule to solve for V_{th} yields

$$V_{\text{th}} = \left(\frac{jX_m}{Z_1 + jX_m}\right) V_1 = 237.699\angle 0.045^\circ\,\text{V}.$$

The corresponding Thévenin impedance is given by

$$Z_{\text{th}} = \left(\frac{1}{Z_1} + \frac{1}{jX_m}\right)^{-1} = \frac{Z_1 jX_m}{Z_1 + jX_m} = 0.0039 + j0.0397\,\Omega.$$

The 3-phase complex power delivered to the grid can be expressed as

$$S_{\text{out}} = 3V_1 {I_1}^*,$$

where I_1 is the stator current. Solving for the reactive power delivered to the grid, we have

$$Q_{\text{out}} = \Im\left\{3V_1 {I_1}^*\right\} = -1.149\,\text{MVAr},$$

where the negative sign indicates that the machine is consuming reactive power. Alternatively, we could solve for the reactive power consumed by the induction machine in a more direct way by

$$Q_{\text{con}} = 3|I_1|^2 X_1 + 3|I_2|^2 X_2 + 3|I_m|^2 X_m = 1.149\,\text{MVAr},$$

where $Q_{\text{out}} = -Q_{\text{con}}$.

Problem 6. (Bonus for undergraduate students, must attempt for graduate students) Look at the dynamical model of the induction generator in Fig. 6.23 in the textbook. Write the input/output relationship between the mechanical torque T_m and the developed torque T_d.

Solution. The main difficulty of this problem is there is a product in the block diagram and the other signals are best expressed using the Laplace transform. We can use the fact that a multiplication in the time-domain is a convolution in the s-domain. Then the blocks to the left hand side of T_d gives

$$T_d = 1.5 L_{12} pp (I_{d2} * I_{q1} - I_{d1} * I_{q2})$$

where

$$\begin{bmatrix} I_{d1} \\ I_{d2} \\ I_{q1} \\ I_{q2} \end{bmatrix} = L_s^{-1} B^{-1} V_{abc}$$

and $*$ is the convolution operator. The right hand side of T_d gives

$$T_m - T_d = \frac{1}{2H}\frac{1}{s}(\omega_s - \omega_2)$$

The overall transformation can be solved by equating the two sides.